

# **3D Broadband Propagation in a Fluctuating Shallow Water Environment**

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Award #: N0001496WX30305

## **LONG-TERM GOAL**

The long-term goal of this project is to provide a numerically efficient and robust acoustic model for the propagation of waves in a three-dimensional fluctuating shallow water environment.

## **OBJECTIVES**

The scientific objectives of this work, as a first step toward the long-term goal, are:

- 1: Develop a new, more efficient 2D propagation model based on the coupled mode method which is more efficient but as accurate as the parabolic equation (PE) model.
- 2: Use this model to study how signal coherence in the ocean is affected by sound speed fluctuations caused by internal waves.

## **APPROACH**

Coupled mode equations are generally derived from the wave equation by dividing a range dependent waveguide into range independent sections where in each section the wave equation can be solved as a sum of local normal modes [1]. The coupling between modes are expressed as integrals with respect to depth of the local modes and their range derivatives.

The use of the wave equation in deriving the coupled mode equations results in two coupling matrices: one which involves first and another which involves second derivatives of the local modes with respect to range. Computation of the coupling matrices, particularly the one which involves the second derivative of the modes with respect to range, is impractical and inaccurate at best. It is therefore common to resort to approximations, which, among other things, destroy the anti-symmetry of the coupling matrices [2], [3], [4]. An energy conserving solution requires the coupling matrices to be anti-symmetric.

Instead of using the wave equation, our approach is to use the equations of motion to derive the coupled mode equations. By this method only one coupling matrix is obtained. What is more important is that this coupling matrix is simple to compute since it involves the depth derivatives rather than the range derivatives of the local modes. Furthermore, this method guarantees energy conservation since the coupling matrix obtained this way is anti-symmetric.

## **WORK COMPLETED**

1. Developed an energy-conserving coupled mode propagation model using the equations of motion.

Report Documentation Page				Form Approved OMB No. 0704-0188	
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1. REPORT DATE <b>30 SEP 1999</b>		2. REPORT TYPE		3. DATES COVERED <b>00-00-1999 to 00-00-1999</b>	
4. TITLE AND SUBTITLE <b>3D Broadband Propagation in a Fluctuating Shallow Water Environment</b>				5a. CONTRACT NUMBER	
				5b. GRANT NUMBER	
				5c. PROGRAM ELEMENT NUMBER	
6. AUTHOR(S)				5d. PROJECT NUMBER	
				5e. TASK NUMBER	
				5f. WORK UNIT NUMBER	
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) <b>SPAWAR Systems Center San Diego,53560 Hull Street,San Diego,CA,92152</b>				8. PERFORMING ORGANIZATION REPORT NUMBER	
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)				10. SPONSOR/MONITOR'S ACRONYM(S)	
				11. SPONSOR/MONITOR'S REPORT NUMBER(S)	
12. DISTRIBUTION/AVAILABILITY STATEMENT <b>Approved for public release; distribution unlimited</b>					
13. SUPPLEMENTARY NOTES					
14. ABSTRACT					
15. SUBJECT TERMS					
16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT <b>Same as Report (SAR)</b>	18. NUMBER OF PAGES <b>6</b>	19a. NAME OF RESPONSIBLE PERSON
a. REPORT <b>unclassified</b>	b. ABSTRACT <b>unclassified</b>	c. THIS PAGE <b>unclassified</b>			

2. Programmed and tested this model by applying it to compute the acoustic field in two simulated range dependent environments, one involving propagation over a sloping bottom and the other involving propagation through internal waves.
3. A journal paper containing the derivation of the model and its application is currently in progress.
4. In collaboration with the Marine Physical Laboratory [5] this model is currently being applied to compute the acoustic field in the SWARM-95 [6] experiment.

## RESULTS

*The coupled mode model:*

To derive the coupled mode equation, we start with the equations of motion [7],

$$\begin{aligned}\nabla p &= \rho \omega^2 \mathbf{u} \\ \partial_x u_n &= -\frac{p}{\rho c^2} - \hat{z} \cdot \partial_z \mathbf{u} \\ [p]_+^+ &= 0, \quad [\hat{n} \cdot \mathbf{u}]_+^+ = 0\end{aligned}$$

where in the above  $\mathbf{u}$  is the displacement vector and  $p$  is the pressure. The bottom equation represents the interface and boundary conditions for the field quantities. The first step in deriving the coupled mode equations is to express the pressure and displacement in terms of the local modes with coefficients that are a function of range:

$$\begin{aligned}p &= \sum_n c_n p_n e^{ik_n x} \\ \mathbf{u} &= \sum_n c_n \mathbf{u}_n e^{ik_n x}\end{aligned}$$

Substituting these expressions in the equations of motion, using the interface conditions and the orthogonality of the modes, after considerable manipulations we obtain,

$$2\partial_x c_m k_m + c_m \nabla_x \cdot \mathbf{k}_m = \sum_{n \neq m} A_{mn} c_n$$

In the above equations  $x$  represents the direction along range,  $k_n$  is the wavenumber and the coupling coefficient matrix is given by

$$\begin{aligned}(k_m - k_n) A_{mn} &= \left\{ \int_0^B \left[ \partial_x \left( \frac{1}{\rho} \right) \left[ (k^2 - k_m k_n) p_m p_n - \partial_z p_m \partial_z p_n \right] + \partial_x k^2 \frac{p_m p_n}{\rho} \right] dz \right. \\ &\quad \left. + \left[ \frac{1}{\rho} \partial_z p_m \partial_z p_n + \frac{1}{\rho} (k^2 - k_m k_n) \right]_+^+ \partial_x h \right\} e^{i(k_n - k_m)x}\end{aligned}$$

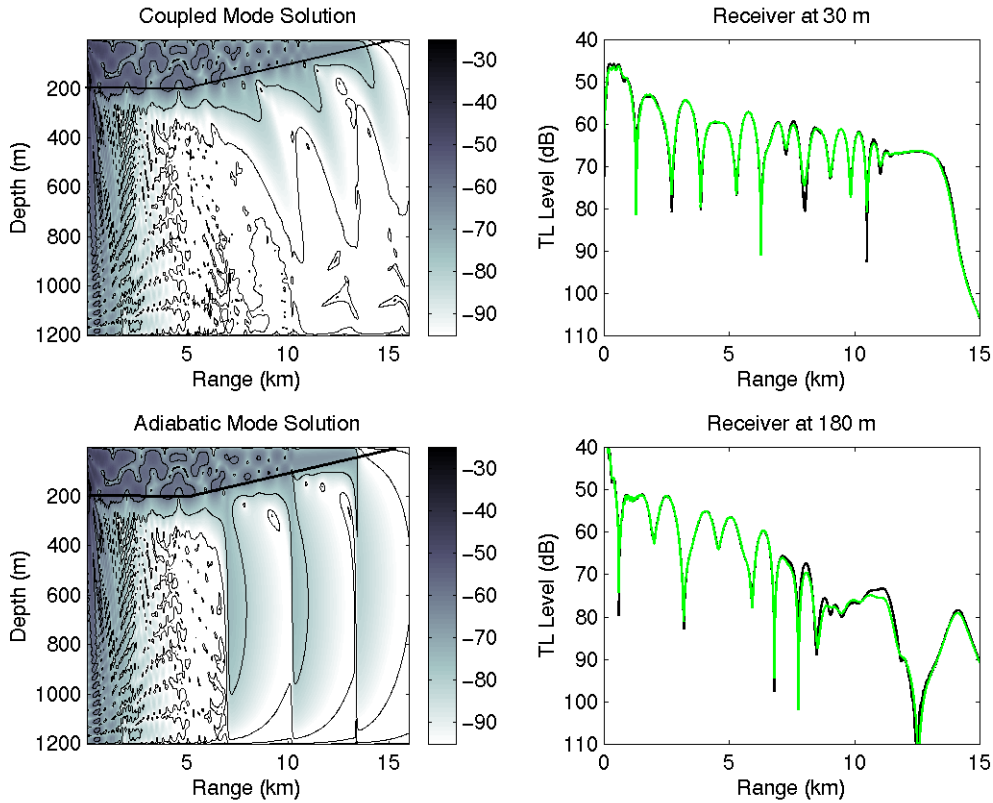
In the above equation the first term represents mode coupling due to variations in sound speed and density and the second term represents mode coupling due to variations in water depth as a function of

range. This model has three remarkable properties: First, it contains only one coupling matrix. Second, the coupling matrix can be computed from the local modes and their depth derivatives (it does not involve range derivatives). And finally the coupling matrix is anti-symmetric in exchanging mode  $m$  with mode  $n$  which guarantees energy conservation between modes. These properties along with the fact that the field quantities can be computed from a first order differential equation makes this model very attractive.

The validity of this model was tested by using it to compute the acoustic field in two simulated environments. One involved propagation over a sloping bottom, and the other involved propagation in a flat waveguide where the effects of internal waves were modeled by periodic sound speed perturbations in range. In both cases the results were compared with that of the parabolic equation (PE) [8] and the adiabatic normal mode models.

#### *Propagation over a sloping bottom:*

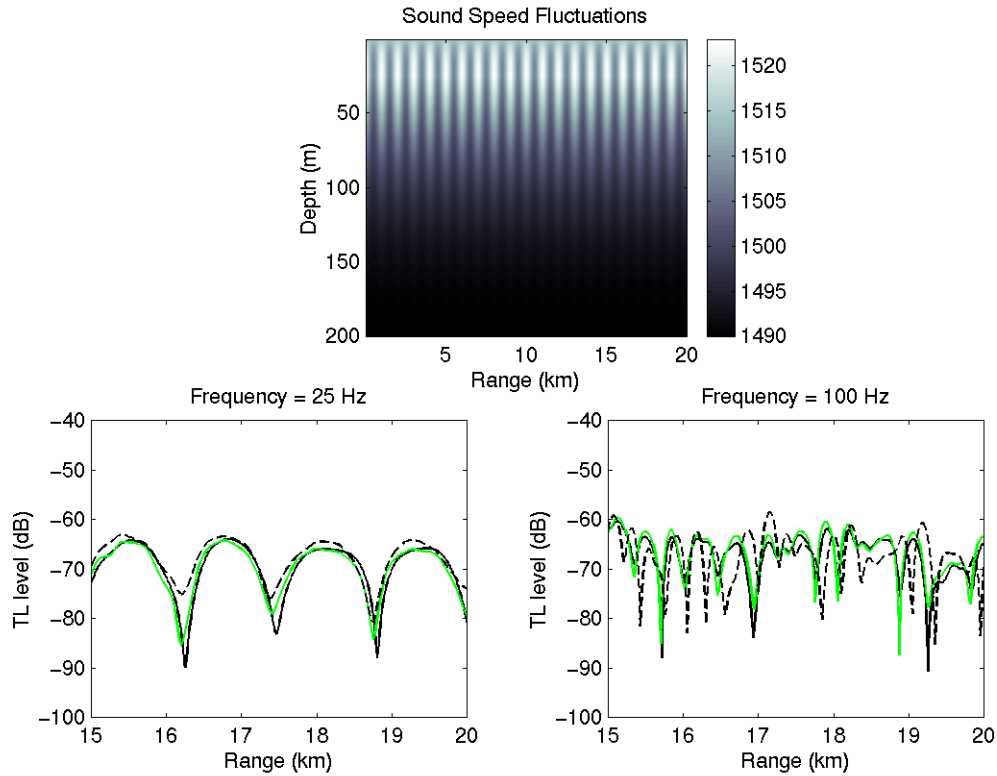
The ocean environment in this case is shown in Fig. 1. It consists of a 5 km flat region where the water depth stays constant at 200 m followed by a sloping region where the water depth decreases to zero in 10 km, giving it a slope of a little over one degree. A 25 Hz source is located at 180m depth. The top left panel in Fig. 1 shows the acoustic field as a function of range and depth computed using this model. The bottom left panel shows the acoustic field computed using the adiabatic mode model. At this frequency there are three propagating modes at 200 m of water. The modes begin to cut off as the water depth decreases. The coupled mode model predicts the correct field behavior in a range dependent waveguide. Observe that in the coupled mode computation, as modes go through cut off they slowly transfer their energy to the bottom by coupling to the bottom modes. This shows that the coupled mode model accounts for mode coupling [9]. In the adiabatic mode computation, where mode coupling is not included, modes abruptly disappear into the bottom as they go through cut off. For a quantitative comparison of the coupled mode model with the PE model, transmission loss as a function of range was plotted for two receiver depths. These are shown in the two right panels of Fig. 1. The coupled model shows excellent agreement with the PE model.



**Figure 1:** The left two panels show the acoustic field as a function of range and depth from a source at 180m depth computed using the coupled mode model (top left) and the adiabatic mode model (bottom left). Note that the coupled mode model predicts the correct field behavior near cut off. The right two panels show a comparison in transmission loss between the PE model (black) and the coupled mode model (gray) for two receiver depths in the same environment.

#### *Propagation through internal waves:*

In this case the ocean environment consists of a flat waveguide 20 km long where the spatial sound speed variations due to internal waves is modeled by periodic sound speed perturbations of  $\pm 7$  m/sec. The transmission loss was computed for a source placed at 30 meters operating at two different frequencies, 25 Hz and 100 Hz, using the coupled mode model, the PE model and the adiabatic mode model. The results are shown in Fig. 2. As can be seen in Fig. 2, there is good agreement between the PE model and the coupled mode model for both frequencies. However, the adiabatic mode model does not predict the field behavior accurately even at 25 Hz. Since mode coupling is proportional to the square of frequency, the results obtained from the adiabatic mode model, which does not account for mode coupling, are expected to be even less accurate at high frequencies. This is evident in the transmission loss results for the 100 Hz case.



**Figure 2:** The top panel shows the internal waves perturbations used in this simulation. The bottom panels show transmission loss as a function of range for a receiver depth of 35 meters computed using the PE (black), the coupled mode (gray) model and the adiabatic mode (dashed) models. The coupled mode model agrees very well with the PE model for both frequencies. Due to strong mode coupling, the adiabatic mode model does not predict the field behavior accurately at 100 Hz.

## IMPACT/APPLICATION

This model is particularly appropriate for use in tomography and generating replica field vectors used in matched field processing (MFP) work sponsored by ONR and DARPA.

## RELATED PROJECTS

Under the sponsorship of the Internal Research (IR) program at the SPAWAR Systems Center, San Diego, the above coupled mode method is being extended to model propagation in a range dependent elastic medium.

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